MTH 1420, SPRING 2012
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SECTION 5.3: DEFINITE INTEGRALS

HW: 4, 11, 18, 25, 31, 44

Practice: 5, 8, 15, 19, 23, 39, 42 (no need to graph), 46[1], 53

1. Introduction

In the previous sections we talked about how to approximate the area under a curve, and we also discussed a method of using a limit to calculate the *actual* area under a curve (as opposed to just an approximation). In this section we give a much faster method for calculating the area under a curve, which is known as integration.

2. Indefinite Integrals

In Calculus 1 we discuss the operation of taking an *antiderivative* of a function. We now present a notation for this.

Definition 1. We say that a function $F(x)$ is an **indefinite integral** of a function $f(x)$ if $F'(x) = f(x)$. We write

$$F(x) = \int f(x)dx$$

The "dx" is a notational place holder and does not have any bearing on the calculation. Note that for a given function f there are an infinite number of indefinite integrals (that is, antiderivatives).

Example 2. Calculate the general indefinite integral $\int(x^3 + 3x^2)dx$.

Exercise 3. Calculate the general indefinite integrals for the following:

(a) $\int \sin x \, dx$

(b) $\displaystyle\int \sqrt[4]{x^7} dx$

(c) $\int \csc x \cot x \, dx$

(d) $\displaystyle\int e^x + \frac{-1}{\sqrt{1-x^2}} dx$

(e) $\displaystyle\int (\ln 6) 6^x \, dx$

(f) $\displaystyle\int \frac{x^7 + x^3 + 1}{x^4} dx$

These last two are tricky:

(g) $\int -\csc^2(10x) dx$

(h) $\int (5x^4 \sin x + x^5 \cos x) dx$

3. Definite Integrals

A *definite integral* is similar to an indefinite integral notation-wise (it just has limits of integration). This is because indefinite integrals can be used to measure the area under a curve.

Definition 4. If f is continuous on an interval $[a,b]$, the definite integral of f from a to b is

$$\int_a^b f(x)dx = F(b) - F(a)$$

where F is *any* indefinite integral (aka antiderivative) of f (that is, any F such that $F' = f$).

Example 5. Calculate the definite integral $\displaystyle\int_2^4 (x^3 + 3x^2)dx$. Use two different antiderivatives to see that you get the same answer.

Exercise 6. Calculate the definite integral $\displaystyle\int_0^2 x^3\, dx$. Compare this answer to Examples 3 and 4 from Section 5.1 when you are finished.

Theorem 3.1. *The definite integral $\int_a^b f(x)dx$ measures the area between the curve f and the x-axis in the interval $[a,b]$, as long as that area is positive. If f is below the x-axis then the definite integral will give you the negative of the area, and if f crosses the x-axis in the interval then the definite integral gives you the difference between the areas above and below the x-axis (that is, the positive area above plus the negative area below).*

An example that illustrates the difference between the definite integral and the area under a curve is as follows:

Example 7. The velocity function for a particle on a line is $v(t) = t^2 - 2t - 8$ meters/second, for $1 \leq t \leq 6$. Find (a) the displacement and (b) the total distance traveled.

Example 8. Calculate the definite integral $\displaystyle\int_0^{\pi/3} \frac{\sin\theta + \sin\theta\tan^2\theta}{\sec^2\theta}\,d\theta$.

Exercise 9. Evaluate $\displaystyle\int_1^8 \sqrt[3]{x}\, dx$.

Exercise 10. Evaluate $\displaystyle\int_0^\pi (e^x - \sin x) dx$.

Exercise 11. Consider the curve $f(x) = x^2(x-3)$. Calculate (a) the definite integral for f from 0 to 4, as well as (b) the area between f and the x-axis from 0 to 4. Note: you should get a different answer for (a) and (b).

Exercise 12. A honeybee population starts with 100 bees and increases at a rate of $n'(t)$ bees per week. What does

$$100 + \int_0^{15} n'(t)dt$$

represent?

Notes

[1]Answers for evens: #8: 10e, #42: $e^x - (2/3)x^3 + c$, #46:$\tan t + \sec t + C$